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CP and T Violation in Neutrino Oscillations and Invariance of Jarlskog's Determinant to Matter Effects

P. F. Harrison

Physics Department, Queen Mary and Westfield College
Mile End Rd. London E1 4NS. UK ¹

and

W. G. Scott

Rutherford Appleton Laboratory
Chilton, Didcot, Oxon OX11 0QX. UK ²

Abstract

Terrestrial matter effects in neutrino propagation are T -invariant, so that any observed T violation when neutrinos pass through the Earth, such as an asymmetry between the transition probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\nu_e \rightarrow \nu_\mu)$, would be a direct indication of T violation at the fundamental level. Matter effects *do* however modify the magnitudes of T -violating asymmetries, and it has long been known that resonant enhancement can lead to large effects for a range of plausible values of the relevant parameters. We note that the determinant of the commutator of the lepton mass matrices is invariant under matter effects and use this fact to derive a new expression for the T -violating asymmetries of neutrinos propagating through matter. We give some examples which could have physical relevance.

¹E-mail:p.f.harrison@qmw.ac.uk

²E-mail:w.g.scott@rl.ac.uk

Evidence for oscillations of atmospheric neutrinos [1] indicates that at least muon and tau neutrinos participate in lepton mixing with large amplitudes. Other evidence of a deficit of detected solar neutrinos [2] indicates that electron neutrinos must also participate in lepton mixing, and the most recent data [3] disfavour small mixing angles. The participation of all three species of neutrinos in lepton mixing raises the possibility of CP and T violations in neutrino oscillations, and, given the recent evidence of direct CP violation in the hadronic sector [4], it would perhaps be surprising if such effects were not also manifested in the leptonic sector. The emergence of large mixing parameters in the leptonic sector offers the exciting prospect of potentially large CP - and T -violating asymmetries. One possibility [5] which has not been excluded [6] is that CP and T violations are maximal for neutrinos in vacuum. A number of other authors [7] have explored the phenomenology of CP and T violation in neutrino oscillations for several different scenarios of lepton mass and mixing parameters.

In both solar and atmospheric experiments, neutrinos can traverse a significant fraction of the Earth. As long-baseline accelerator and reactor experiments push to still longer baselines, the amount of material traversed by man-made neutrino beams also increases. This makes the understanding of matter effects in neutrino oscillations essential in order to determine the complete pattern of neutrino masses and mixings. The particle/anti-particle non-invariance of the matter term in the effective Hamiltonian for neutrino propagation in matter means that observation of a particle-antiparticle asymmetry does not necessarily indicate a fundamental CP -violation, although it is, in principle, possible to discriminate between the two sources of such effects using the observed distance- and energy-dependence [8]. However, matter effects in neutrino propagation between two points at the surface of the Earth are T -symmetric, and any observed T violation, such as an asymmetry between the transition probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\nu_e \rightarrow \nu_\mu)$ would certainly be a direct indication of T violation (and via CPT invariance, of CP violation) at the fundamental level.

Despite the fact that matter effects in the Earth are T -invariant, they have a non-trivial effect on the signature of fundamental T violations of ν and $\bar{\nu}$ separately, as well as on CP violations, through their influence on the effective neutrino mass and mixing parameters. In fact, it has long been known [9] that very large resonant enhancements of the T -violating asymmetries are possible in terrestrial matter. In this paper, we derive a simple new result for the effect of matter on the T - and CP -violating asymmetries, and explore some of the consequences in experimentally preferred scenarios of mass and mixing parameters.

In the case of three generations of massive neutrinos, in a weak interaction basis which diagonalises the charged lepton mass matrix, $M_\ell = D_\ell$, the neutrino mass ma-

trix, M_ν is in general, an arbitrary 3x3 matrix. The Hermitian square of the neutrino mass matrix, $M_\nu M_\nu^\dagger$, may be diagonalised to find its eigenvalues, and its eigenvectors form the columns of the lepton mixing matrix, U . It is well-known that under these circumstances, neutrinos propagating in vacuum undergo flavour oscillations, and furthermore, in general, these result in CP - and T -violating asymmetries.

The CP - and T -violating asymmetries in the transition probabilities are given (for arbitrary mixing matrix) by the universal function

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) &= P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \\ &= 16J \sin(\Delta_{12}L/2) \sin(\Delta_{23}L/2) \sin(\Delta_{31}L/2) \end{aligned} \quad (1)$$

for any pair of flavour indices α, β , where J is Jarlskog's mixing matrix-dependent invariant [10] and the $\Delta_{ij} = (\lambda_i - \lambda_j)$ are the three differences of eigenvalues of the Hamiltonian, H . In the vacuum case, it is sufficient to set

$$H = M_\nu M_\nu^\dagger / 2E \quad (2)$$

so that $\lambda_i = m_i^2/2E$, where m_i is the mass of the i th neutrino mass eigenstate (as usual, we number the eigenstates in increasing order of mass). We note that the parameter J has a maximal value of $1/(6\sqrt{3})$, and that the product of the three sine functions (the arguments are not independent) has a maximal value of $3\sqrt{3}/8$, which it takes when all three arguments are separated by 120° [11]. The maximum magnitude of the product of three sines is controlled by the smallest of the three arguments, $\Delta_{12}L/2$, and we can consider $2/\Delta_{12}$ as the reduced wavelength of the asymmetry. The asymmetry is observable only once this term has developed a significant phase, and if it is furthermore not averaged to zero by resolution effects. This fact limits the observability of CP and T violations in neutrino oscillations in vacuum to a window of parameter-space, as pointed-out in Ref. [12].

In the case that the neutrino beam passes through matter of uniform density, the Hamiltonian is modified to $H' = H + \Delta H$, where

$$\Delta H = \pm\sqrt{2}GN_e \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (3)$$

The \pm sign is to be taken as $+$ for neutrino and $-$ for anti-neutrino propagation, and makes explicit the particle/anti-particle asymmetry introduced by matter effects. The effect of matter is therefore to modify the mass eigenvalues and the mixing matrix elements, compared with their vacuum values.

The matter electron density, N_e , lies, for the Earth, in the range $0 \lesssim N_e \lesssim 6.2 N_A \text{cm}^{-3}$. The expression, Eq. (1), for the T -violating asymmetries is still valid in the case of

propagation through matter of uniform density, except that the eigenvalues and the parameter J appearing in the expression are modified to their matter values λ'_i and J' respectively. For the CP -violating asymmetry between neutrinos and anti-neutrinos, the situation is made much more complicated by the particle/anti-particle asymmetry of the matter term in the Hamiltonian, and there are terms additional to that on the right-hand side of Eq. (1) [8].

Jarlskog [10] has given an easy way to calculate the CP - and T -violation parameter, J , in terms of the mass matrices and their eigenvalues. This may be written:

$$2\Delta_{12}\Delta_{23}\Delta_{31}J = \text{Im}\{\text{Det}[M_\ell^2, H]\}/(\Delta_{e\mu}\Delta_{\mu\tau}\Delta_{\tau e}) \quad (4)$$

where the $\Delta_{\ell\ell'}$ on the RHS refer to the differences between the squared charged lepton masses. This formula is valid in any weak basis, but it is useful for our purposes to evaluate the RHS in the weak basis which diagonalises the charged lepton mass matrix, in which it can be written:

$$\text{Im}\{\text{Det}[D_\ell^2, H]\}/(\Delta_{e\mu}\Delta_{\mu\tau}\Delta_{\tau e}) = 2\text{Im}(H_{12}H_{23}H_{31}). \quad (5)$$

It is easy to see that use of the vacuum Hamiltonian, H , or the matter-modified one, H' in Eq. (5) leaves the result invariant, as ΔH is diagonal in this basis. In fact, more generally, the commutator

$$[D_\ell^2, H] = [D_\ell^2, H'] \quad (6)$$

is invariant to matter effects in this basis. Taking the determinant on both sides of Eq. (6) yields the physical result, valid in any weak basis, that Jarlskog's determinant of the commutator of the two lepton mass(-squared) matrices is invariant to matter effects, and, from Eq. (4) that:

$$\Delta_{12}\Delta_{23}\Delta_{31}J = \Delta'_{12}\Delta'_{23}\Delta'_{31}J' \quad (7)$$

(where the primed quantities refer to the matter modified Hamiltonian, as above). The invariance, Eq. (7), is a principal result of this paper (seeming not to have appeared previously in the literature). One consequence is that it enables the phenomenology of T violation for neutrinos in matter to be stated in a new and transparent way.

The T -violating asymmetries of Eq. (1) can now be generalised to neutrino propagation through matter of uniform density in the following form:

$$P'(\nu_\alpha \rightarrow \nu_\beta) - P'(\nu_\beta \rightarrow \nu_\alpha) = 16J \frac{\Delta_{12}\Delta_{23}\Delta_{31}}{\Delta'_{12}\Delta'_{23}\Delta'_{31}} \sin(\Delta'_{12}L/2) \sin(\Delta'_{23}L/2) \sin(\Delta'_{31}L/2) \quad (8)$$

where Eq. (7) has been used to rewrite J' in terms of its corresponding vacuum value, J , the vacuum masses, and the matter-modified masses, a form which is completely

independent of the matter-modified mixing matrix elements themselves. This result is exact, and valid for arbitrary vacuum Hamiltonian, ie. for arbitrary neutrino mass and mixing parameters. Eq. (8) shows clearly that in general, both the magnitude, and the wavelength of T -violating asymmetries are modified in matter compared with their vacuum values, and it makes explicit the correlation between these two effects. The modification of the magnitude is by a factor:

$$\mathcal{R} = \frac{J'}{J} = \frac{\Delta_{12}\Delta_{23}\Delta_{31}}{\Delta'_{12}\Delta'_{23}\Delta'_{31}}, \quad (9)$$

which, in general, can be larger or smaller than unity. Provided that the asymmetry wavelength is not increased by matter effects beyond observability, potentially useful enhancements of the magnitude of T violation can occur in matter.

In the form of Eq. (8), a number of results concerning T -violation for neutrinos propagating in matter can be seen rather easily. For example, it is manifest that even in matter, the T -violating asymmetries, as defined here, are independent of flavour. In the low-density limit, $N_e \rightarrow 0$ ($\Delta'_{ij} \rightarrow \Delta_{ij}$ for all i, j), the expression, Eq. (8), reduces to the normal vacuum case, Eq. (1), as expected. Furthermore, in the small- L limit (ie. if the propagation distance is small compared with the scale of the shortest matter oscillation length, $L \ll 2/\Delta'_{13}$), the equation reduces to the vacuum case, Eq. (1), as all the sine functions in Eq. (8) may be approximated by their arguments and all the primed variables cancel. For $L \rightarrow 0$ therefore, there is no observable effect due to matter on T -asymmetries (nor indeed on any observable aspect of the oscillation phenomenon [13]). In general, if $J \neq 0$ and there are no degeneracies in vacuum, Eq. (7) ensures that there are no degenerate eigenvalues in matter ($\Delta'_{12}\Delta'_{23}\Delta'_{31} \neq 0$) and hence that \mathcal{R} remains finite.

We note that for an arbitrary Hermitian matrix, H , the combination of its eigenvalues, $\Delta_{12}\Delta_{23}\Delta_{31}$, is the square-root of the discriminant of its eigenvalue equation:

$$\Delta_{12}\Delta_{23}\Delta_{31} = \sqrt{-4\mathcal{S}^3 + \mathcal{S}^2\mathcal{T}^2 - 27\mathcal{D}^2 + 18\mathcal{D}\mathcal{T}\mathcal{S} - 4\mathcal{D}\mathcal{T}^3} \quad (10)$$

where the invariants \mathcal{T} , \mathcal{S} and \mathcal{D} are respectively the trace, the sum of the principle minors and the determinant of H . This can be further simplified by noting that we have the freedom to add an arbitrary multiple of the unit matrix to H without altering its discriminant, so that we can choose either $\mathcal{T} = 0$ or one of the eigenvalues, $\lambda_i = 0$. So, for any given model of the neutrino masses and mixing angles in vacuum, the magnitudes of the T -violating asymmetries in matter can, in principle, be calculated directly from the vacuum parameters of the model and the (simplified) invariants of the matter-modified Hamiltonian, H' , thereby obviating the need to diagonalise explicitly H' to find the matter-modified mixing angles.

In the general case, application of Eq. (10) to the matter-modified Hamiltonian to calculate the denominator of Eq. (9) yields the square-root of a quartic function of N_e , with coefficients which are functions of the vacuum mixing parameters (masses and mixing angles) and the neutrino energy. The presence of this quartic function means that \mathcal{R} has either one or two resonant maxima, as a function of the matter density. These correspond to the well-known matter resonances, occurring approximately where pairs of matter-modified neutrino masses are most nearly degenerate [13]. Values of \mathcal{R} can be arbitrarily large, if J is sufficiently small (clearly, $J' = RJ$ cannot exceed $1/6\sqrt{3}$). For very large values of N_e , the quartic term dominates, and T asymmetries are suppressed by $\sim 1/N_e^2$.

In the remainder of this paper, we explore some specific examples which could have physical relevance. In order to ensure that our discussion is relevant to experiment, we will restrict our considerations to regions of parameter space which are not excluded by present, corroborated neutrino experiments, such as the Super Kamiokande atmospheric neutrino data [1] and the Super Kamiokande and Gallium solar neutrino data [2]. We will use for the lepton mixing matrix, U , a conventional form in which there are three real mixing angles and one complex phase such that: $U_{e3} = \sin \theta_{13}$, $U_{e2} = \sin \theta_{12} \cos \theta_{13}$, $U_{\mu 3} = \sin \theta_{23} \cos \theta_{13} e^{i\delta}$ and all the other elements are fixed by unitarity.

A priori, our preferred scenario for neutrino oscillations is the threefold maximal, or tri-maximal scheme [5], which still provides a broadly consistent account of all the corroborated sets of experimental data on neutrino oscillations. In this scheme, all the elements of the lepton mixing matrix have equal moduli of magnitude $1/\sqrt{3}$, the vacuum value of J takes its maximal value, $1/(6\sqrt{3})$, and all observed neutrino disappearance data are the result of neutrino oscillations governed by the scale of the larger neutrino mass-squared difference, $\delta m_{13}^2 \sim 10^{-3} \text{ eV}^2$, with δm_{12}^2 unresolved even by the solar data. This scheme is summarised by $\theta_{12} = \theta_{23} = 45^\circ$, $\theta_{13} = \sin^{-1}(1/\sqrt{3})$ and $\delta = 90^\circ$. In this case, \mathcal{R} is always less than unity in matter, so that asymmetries are always suppressed.

A viable, and even experimentally favoured, alternative to tri-maximal mixing effectively factorises the atmospheric and solar scales by setting $U_{e3} \simeq 0$ in the mixing matrix [14]. The atmospheric data then require $\theta_{23} \simeq 45^\circ$, and an energy-independent solar suppression of $1/2$ is obtained by setting $\theta_{12} = 45^\circ$ in the original bi-maximal scheme [14]. We have ourselves proposed [6] (see also [15]) a particular variant of the bi-maximal scheme with $U_{e2} = U_{\mu 2} = U_{\tau 2} = 1/3$ which simulates very effectively tri-maximal mixing with a solar suppression of $5/9$. In bi-maximal-type schemes, $J_{CP} = 0$ and there are no fundamental CP - or T -violating asymmetries involving neutrinos, in

vacuum or in matter. There are still fake $\nu/\bar{\nu}$ asymmetries due to matter effects, but these are of little fundamental interest.

For illustrative purposes, we have explored in this paper an ansatz which interpolates smoothly between tri-maximal and bi-maximal mixing:

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} \cos \theta_{13} & \frac{1}{\sqrt{2}} \cos \theta_{13} & \sin \theta_{13} \\ -\frac{1}{2}(1 + \sin \theta_{13} e^{i\delta}) & \frac{1}{2}(1 - \sin \theta_{13} e^{i\delta}) & \frac{1}{\sqrt{2}} \cos \theta_{13} e^{i\delta} \\ \frac{1}{2}(1 - \sin \theta_{13} e^{i\delta}) & -\frac{1}{2}(1 + \sin \theta_{13} e^{i\delta}) & \frac{1}{\sqrt{2}} \cos \theta_{13} e^{i\delta} \end{pmatrix} \quad (11)$$

with $\delta = 90^\circ$. We have assumed that $\delta m_{13}^2 \sim 10^{-3} \text{ eV}^2$ and have allowed δm_{12}^2 to remain variable. This scheme has the following properties:

- ν_μ and ν_τ are treated democratically
- ν_1 and ν_2 are treated democratically
- in the limit $\sin^2 \theta_{13} \rightarrow 1/3$, we obtain tri-maximal mixing
- in the limit $\sin^2 \theta_{13} \rightarrow 0$, we obtain bi-maximal mixing
- $J_{CP} = (1/4) \sin \theta_{13} \cos^2 \theta_{13}$ varies between its minimal and maximal values, as we move from bi-maximal to tri-maximal mixing.

For arbitrary $\sin \theta_{13}$ ($0 < \sin \theta_{13} < \frac{1}{\sqrt{3}}$), there is always a resonance where T -violating asymmetries are maximised and this can be probed by choosing the neutrino energy, E , or the density of matter traversed, appropriately, as long as there is sufficient pathlength for the asymmetry to develop. The values of neutrino energy and/or matter density at resonance, and the maximum magnitude of the asymmetry there depend on $\sin \theta_{13}$ and the vacuum masses.

Figs. 1a and 1b show some examples of the maximum magnitude of the T -violating asymmetry defined in Eq. (8), as a function of the matter density in units of $N_A \text{ cm}^{-3}$ for several values of $\sin \theta_{13}$ in this ansatz. The two figures differ in terms of the hierarchy of vacuum mass values used, and illustrate cases with one and with two resonant densities respectively. All the curves are for a fixed neutrino energy, $E = 1.5 \text{ GeV}$, which turns out to be the most suitable energy scale to maximise these effects in terrestrial matter within this model (see Eq. (13)). The Earth's mantle has an electron density of approximately $2N_A \text{ cm}^{-3}$ and the core, roughly $6N_A \text{ cm}^{-3}$. The negative density parts of the curves correspond to anti-neutrinos propagating in normal matter with electron density $|N_e|$. We note that enhancements for neutrinos, for example, are typically compensated by suppressions for anti-neutrinos, and/or for neutrinos at different energies. As shown in Ref. [6], the electron neutrino begins to decouple

completely in the high energy limit, and T asymmetries are suppressed accordingly. The magnitudes of the asymmetries in vacuum can be read from the curves at the zero density point. Several other features are typical of the generic MSW-like matter effect, eg., the resonance gets sharper and the asymmetry maximum gets closer to unity as the vacuum mixing angle decreases.

It is interesting to consider under what circumstances the asymmetry becomes maximal in this ansatz. Even in the limit $\sin \theta_{13} \rightarrow 0$ ($\sin \theta_{13} \neq 0$), where T - and CP -violating asymmetries in vacuum become arbitrarily small, they can reach their maximum possible value of unity in matter. We find that this can be achieved if

$$\delta m_{12}^2 = 2 \cos \theta_{13} \frac{\sqrt{2} - \cos \theta_{13}}{1 + \sin^2 \theta_{13}} \delta m_{13}^2 \quad (12)$$

and then the matter density and neutrino energy at resonance are related by:

$$\sqrt{2} G N_e = \frac{1}{\sqrt{2}} \frac{(1 - 3 \sin^2 \theta_{13})(\sqrt{2} - \cos \theta_{13})}{1 + \sin^2 \theta_{13}} \delta m_{13}^2 / 2E. \quad (13)$$

Under the above two conditions, threefold maximal mixing is achieved at resonance. Eq. (12) implies that for such mixing at resonance, the two Δm^2 values must be of the same order of magnitude, which is probably not ruled out by the present data.

Fig. 2 shows an example of how one of the asymmetries in Fig. 1 develops with propagation distance, in units of 1000 km. It is plotted for neutrinos in matter with a constant density of $1.9 N_A \text{ cm}^{-3}$, ie. close to the density of the Earth's mantle. The matter-enhanced asymmetry is compared with the vacuum asymmetry and the corresponding matter-enhanced asymmetry for anti-neutrinos. The enhanced asymmetry does not develop to the 100% level within the distance scale of the Earth's diameter in this case, (it would, given a longer pathlength), although there are cases where it does. It does however exceed considerably the vacuum asymmetry, along almost the whole trajectory within the Earth, and may be observable when the vacuum asymmetry would not be.

While Eq. (12) may not be satisfied exactly in nature, there is a significant range of parameter space over which large enhancements are possible. We have found the conditions under which the enhancements are maximal, and here, at least for some values of the parameters, matter can induce a situation where the mixing matrix is arbitrarily close to the tri-maximal form. Such a scenario might well be obtained in nature, and we recommend that large T - and CP -violating asymmetries be searched for in future long-baseline neutrino experiments. It may even be possible to site experiments to exploit the matter effects in the Earth so as to maximise the asymmetries.

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Figure Captions

Figure 1. Examples of resonant enhancement and suppression of T violation for neutrino oscillations in matter using the ansatz of Eq. (11). The maximum magnitude of the T -violating asymmetry is plotted as a function of matter density and $\sin \theta_{13}$, for neutrino energy, $E = 1.5$ GeV, with a). $\delta m_{13}^2 = 1.0 \times 10^{-3}$ eV² and $\delta m_{12}^2 = 0.7 \times 10^{-3}$ eV²; b). $\delta m_{13}^2 = 1.0 \times 10^{-3}$ eV² and $\delta m_{12}^2 = 0.2 \times 10^{-3}$ eV². In each case, the point marked “o” represents the vacuum value of the asymmetry for the same vacuum mass parameters. The Earth’s mantle has an electron density of approximately $2N_A$ cm⁻³ and the core, roughly $6N_A$ cm⁻³. NB. The density scale can be converted to a scale of energy in GeV for neutrinos propagating in the Earth’s mantle by multiplying the numbers by ~ 0.75 .

Figure 2. An example of resonant enhancement of T violation for neutrino oscillations in matter. The solid curve shows the T -violating asymmetry as a function of propagation distance, for neutrinos of energy, $E = 1.5$ GeV, with $\delta m_{13}^2 = 1.0 \times 10^{-3}$ eV², $\delta m_{12}^2 = 0.7 \times 10^{-3}$ eV² and $\sin \theta_{13} = 0.1$. The matter electron density in this example is $1.9N_A$ cm⁻³. The dashed line shows the same quantity in vacuum for the same vacuum parameters, and the dotted line shows the same quantity for anti-neutrinos.



